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# Jet and Rocket Propulsion

## AE4451

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### LECTURE 23

# Overview

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- what we saw in Lecture 21:
  - solid rocket motors
    - basic configuration, components, key characteristics
    - burn/regression rate and mass balance
- what we saw in Lecture 22:
  - in-class review of pre-midterm examples
- today:
  - solid rocket motors
    - grain geometry considerations, practical examples
    - calculation methods for pressure history

# Solid-rocket motor (SRM)

## Pressure histories

- motor designer can adjust pressure profile (“history”) of a solid motor by arranging how burning area changes with time (**grain geometry**)
- thrust given by  $\tau = p_o A_t c_\tau$ 
  - i.e. thrust history of motor essentially follows motor’s pressure history
- characterize pressure/thrust histories as generally
  - **progressive**: burn surface increases with time
  - **neutral**: constant with time
  - **regressive**: decrease with time
  - combinations
- internal ballistics analysis: characterizes time history of pressure

We can define a **web distance**  $w_t$   
 = linear amount of propellant consumed as measured **normal** to local burn surface

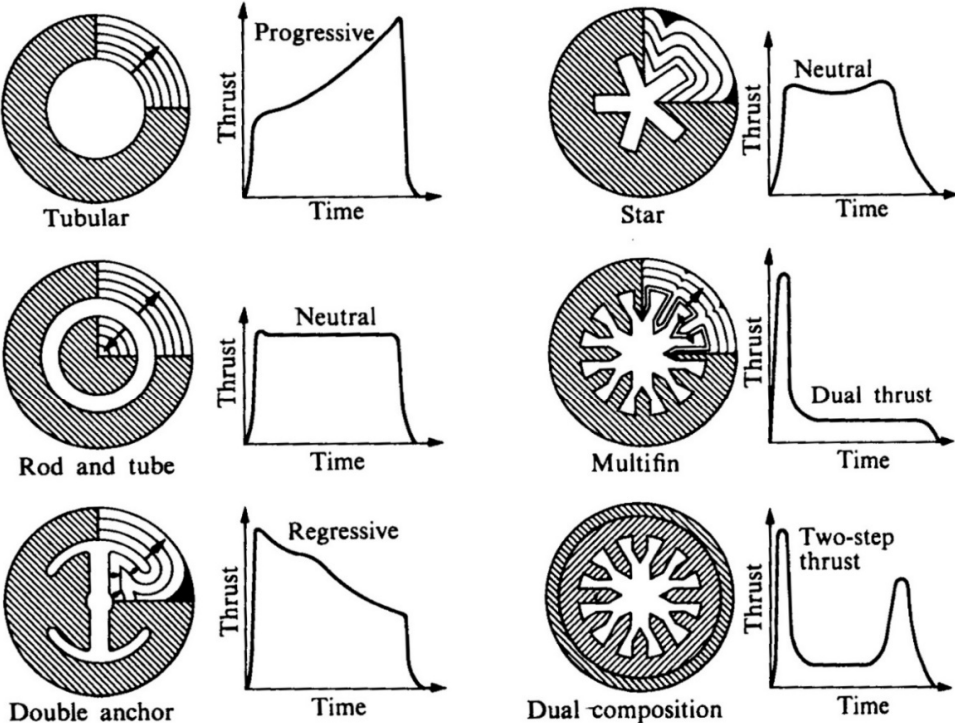
- integral of burning rate history, i.e.

$$w_t = \int_0^{t_b} r(t) dt$$

# Solid-rocket motor (SRM)

## Grain geometries and thrust history

- using grain geometry to modify thrust history



**FIGURE 12.17** Internal-burning charge designs with their thrust-time programs. (Courtesy Shafer [18].)

*from Hill and Peterson*

# Solid-rocket motor (SRM)

## Other grain geometries

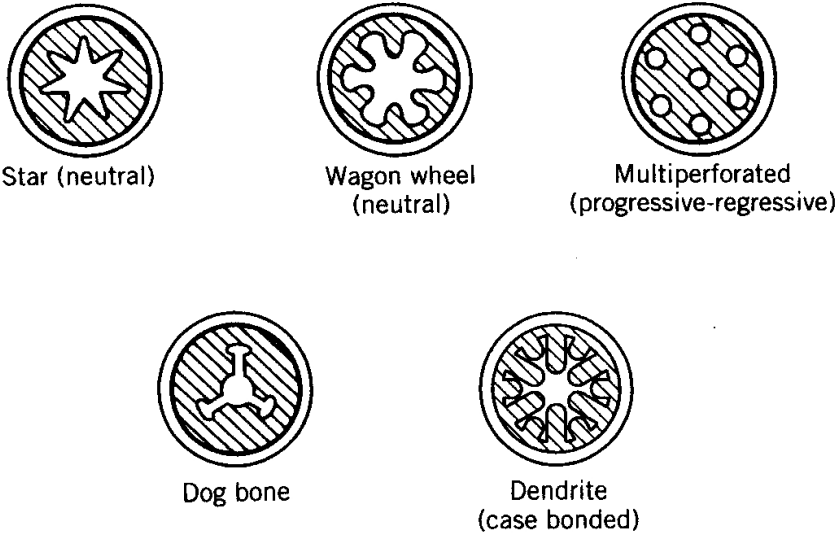
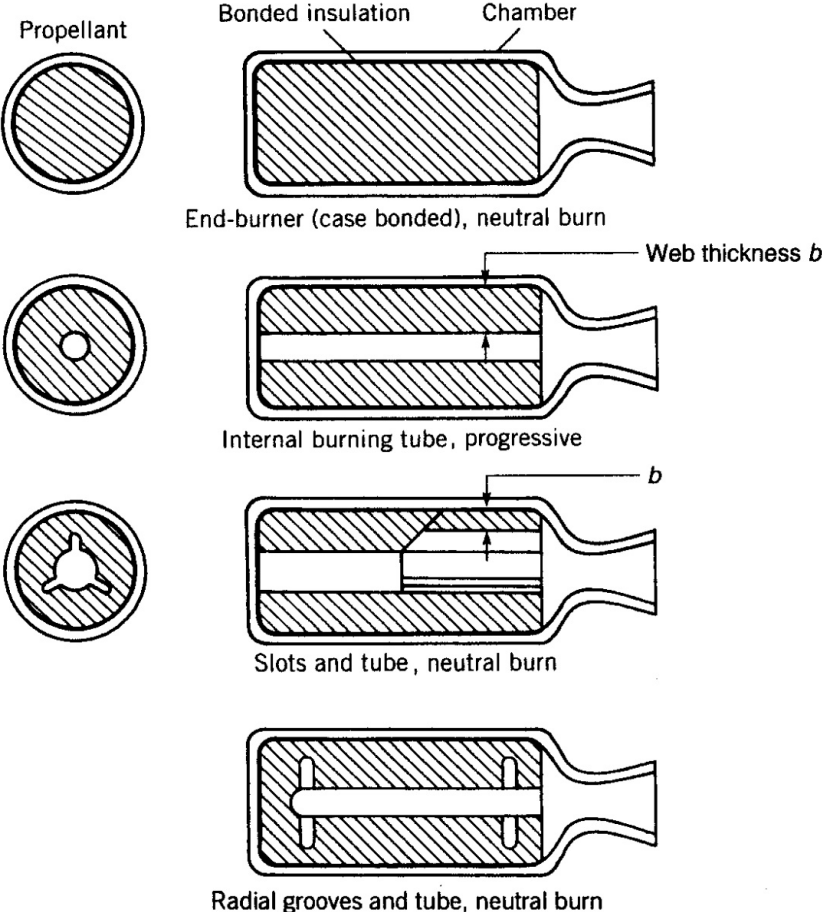


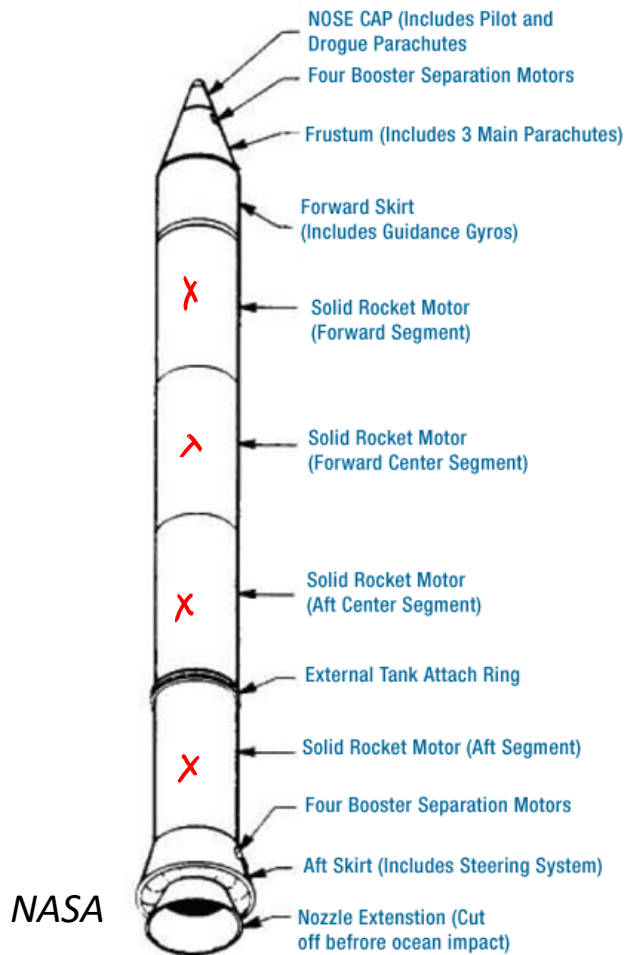
FIGURE 11-16. Simplified diagrams of several grain configurations.



from Sutton

# Solid-rocket motor (SRM)

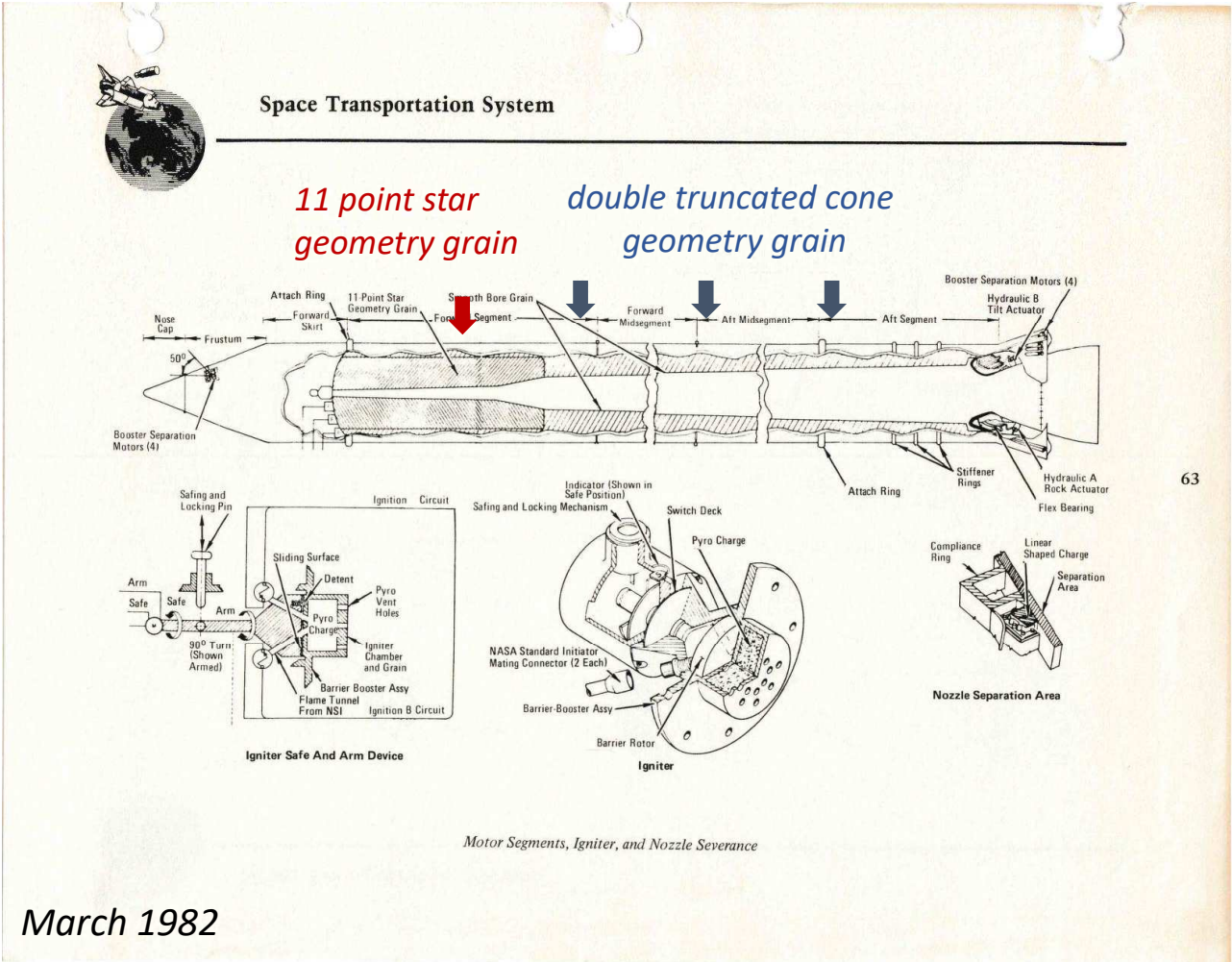
## Space shuttle rocket booster



- largest SRM flown and first designed for reuse
  - diameter = 3.71 m, length = 45.5 m
- sea level thrust: 14.7 MN
- mass:  $5.9 \times 10^5$  kg (inert:  $8.7 \times 10^4$  kg)
  - ~ 71% of thrust at lift-off and ascent
- propellant composition (mass fractions)
  - AP: 69.6%, Al: 16%, *ammonium perchlorate = AP*
  - $\text{Fe}_2\text{O}_3$  (catalyst): 0.4%,
  - HTPB (binder): 12.04%
  - epoxy (curing agent): 1.96%
- four segments
  - 11 point star (neutral) in forward segment
  - double truncated cone (regressive) in 3 segments

# Solid-rocket motor (SRM)

## Space shuttle rocket booster



63

# Solid-rocket motor (SRM)

## Calculation approaches in internal ballistics analysis

- 3D computational fluid dynamics: time-intensive
- 2 approaches to simplify analysis

### lumped-parameter method

- valid for:  $v \ll a$
- only small pressure variations within chamber
- single pressure attributed to entire chamber
- if steady: no mass accumulation in chamber
- can be unsteady

$$\frac{dm}{dt} = 0$$

### ballistic element method

- valid for:  $v > a$
- large pressure variations within chamber: spatial dependence
- single pressure attributed to entire chamber
- split bore into elements
  - apply 1D mass and momentum conservation
  - 2D, 3D effects from empirical calculations

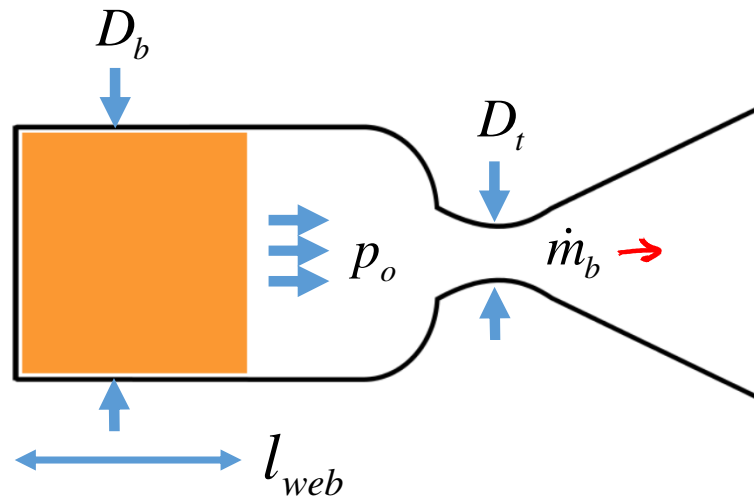
# Solid-rocket motor (SRM)

## Design issues

- typical requirements
  - thrust  $\tau(t)$
  - burn time  $\Delta t_b$  or total impulse  $I_{tot}$
  
- design variables
  - propellant composition (affects e.g.  $c^*$ )
  - grain geometry
  - motor geometry: diameter, length
  - nozzle geometry:  $\epsilon$ , throat diameter
  
- unsteady variables
  - chamber pressure, mass flow rate
  
- other considerations
  - high volume loading fraction ( $V_{propellant}/V_{chamber}$ )
  - low residual propellant mass
  - structural integrity
  - limit erosive burning
  - limit max operating pressure

# Solid-rocket motor (SRM)

## Case study: design of an end-burning motor



- end-burning motor
  - simple geometry
  - constant thrust
  - application: small motors, gas generators

### requirements

$$\Delta t_b = 100 \text{ s}$$

$$\tau_{vac} = 50 \text{ kN}$$

*thrust in vacuum*

### constraints

$$p_o = 4 \text{ MPa}$$

$$\text{nozzle: } c_\tau = 1.85 \quad (\epsilon \sim 30-50)$$

propellant:

$$c^* = 1500 \text{ m/s}, \quad \gamma = 1.2, \quad MW = 24$$

$$\rho_s = 1800 \text{ kg/m}^3$$

$$r = 0.40 [p_o (\text{MPa})]^{0.3} \text{ cm/s}$$

*pressure*

### design variables

$$D_t, D_b, l_{web}$$

assume axisymmetric cylindrical geometry



# Solid-rocket motor (SRM)

## Case study: design of an end-burning motor

- motor diameter  $D_b$

in Lec. 21, we derived for  $\frac{dp_o}{dt} = 0$

$$p_o = r \frac{A_b}{A_t} (\rho_s - \rho_o) c^*$$

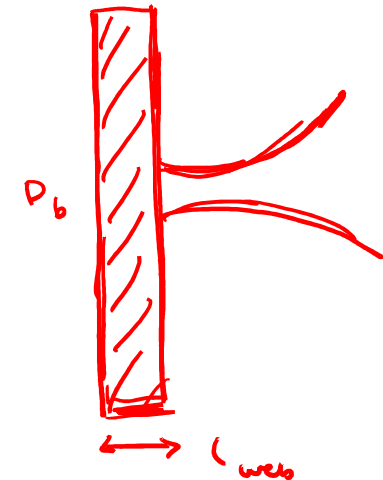
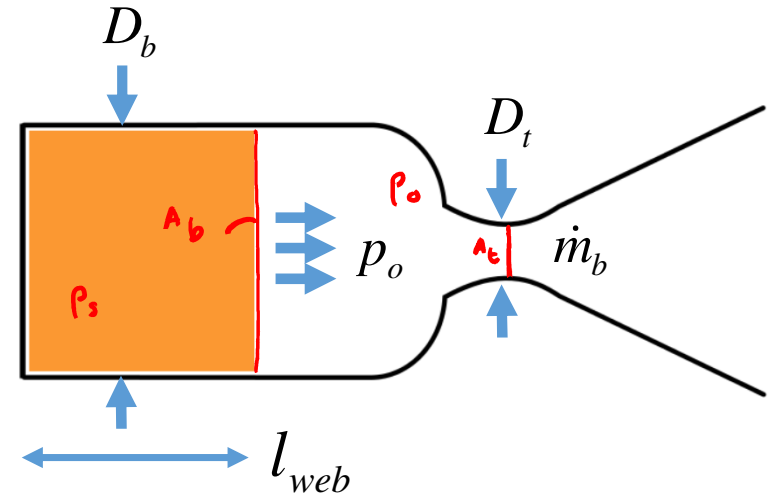
$$\frac{A_b}{A_t} \equiv K = \frac{p_o}{r(\rho_s - \rho_o) c^*} \approx \frac{p_o}{r \rho_s c^*} \quad p_s \gg p_o$$

$$= \frac{4 \times 10^6 \text{ N/m}^2}{0.0061 \text{ m/s} (1800 \text{ kg/m}^3) 1500 \text{ m/s}} = 243$$

$$\Rightarrow D_b = 4.57 \text{ m}$$

$$\Rightarrow D_b / l_{web} \approx 7.5$$

i.e. huge end-burning motors needed to produce high thrust



# Solid-rocket motor (SRM)

## Case study: design of an ~~internal~~<sup>end</sup>-burning motor

- what is the effect of increasing pressure?

$$A_t = \frac{\tau}{p_o c_\tau} \quad : \text{decrease } A_t, \text{ decrease } D_t$$

for constant thrust,  $c_\tau$

$$p_o = r \frac{A_b}{A_t} (\rho_s - \rho_o) c^* \quad : \text{increase } r$$

$$l_{web} = r t_b \quad : \text{increase } l_{web}$$

for constant  $A_b/A_t$ , densities

$$\frac{A_b}{A_t} \equiv K = \frac{p_o}{r(\rho_s - \rho_o) c^*} \cong \frac{p_o}{r \rho_s c^*} \quad : \text{increase } A_b/A_t, \text{ increase motor diameter}$$

for constant  $r$ , densities,  $c^*$

# Solid-rocket motor (SRM)

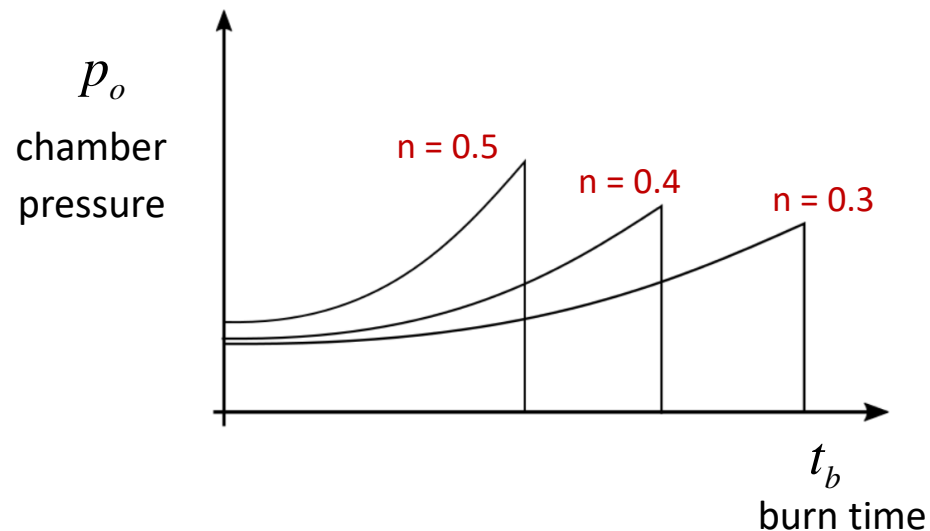
## Effect of changing $n$ , pressure exponent/burning rate exponent

- from previous lecture,

$$\dot{m}_{incr} = A_b (\rho_s - \rho_o) r \propto \underline{p_o^n}$$

- stability when:

- $n \leq 1$ : normally use  $0.3 < n < 0.7$
- $n > 1$ : explosive behavior possible if there's small perturbation in chamber



- changing  $n$  changes the pressure history
  - increase  $n$  increases  $r$
  - increases pressure rise and shortens burning time
- total pressure integral constant

in the example we just saw, we considered an  $n$  of 0.3

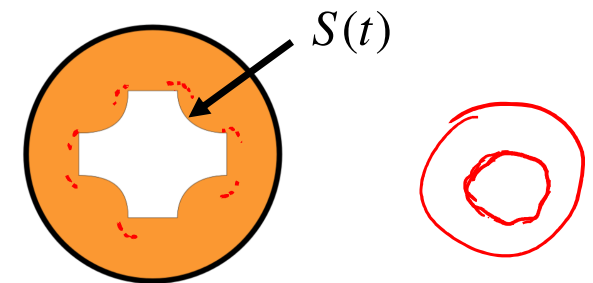
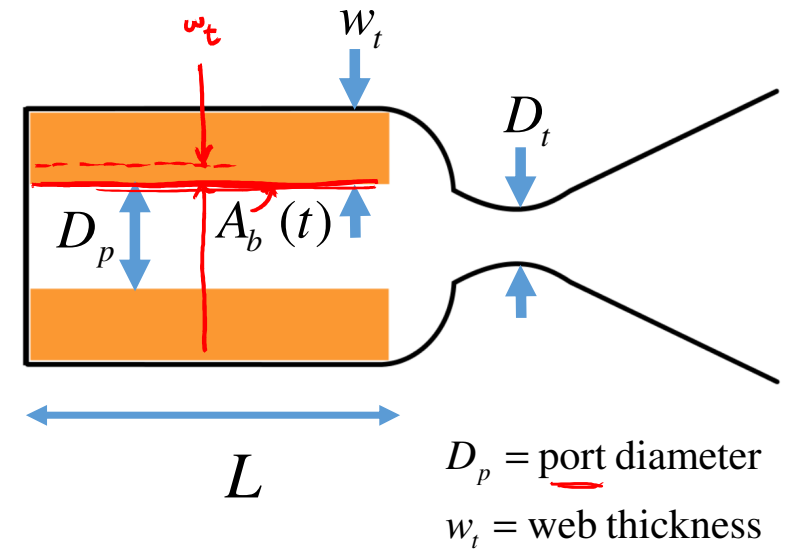
# Solid-rocket motor (SRM)

## Case study: design of an internal-burning motor

- the burn time  $\Delta t_b$  is a function of the web thickness  $w_t$
- length impacts burn area
 
$$A_b(t) = L \times S(t)$$

$$S(t) = \text{perimeter}$$
- for given initial grain geometry, need to know how  $S$  evolves with time

- integration of burning surface location due to regression acting normal to surface



# Solid-rocket motor (SRM)

## Case study: design of an internal-burning motor

- typical requirements

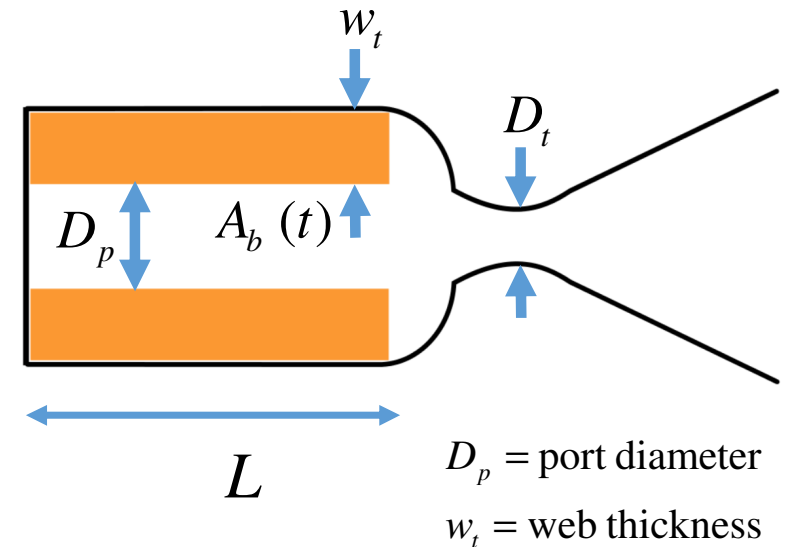
$$\Delta t_b \quad \tau(t)$$

- unsteady parameters

$$p_0(t), \dot{m}(t) \dots$$

- design variables

$$D_t, A_b(t), L$$



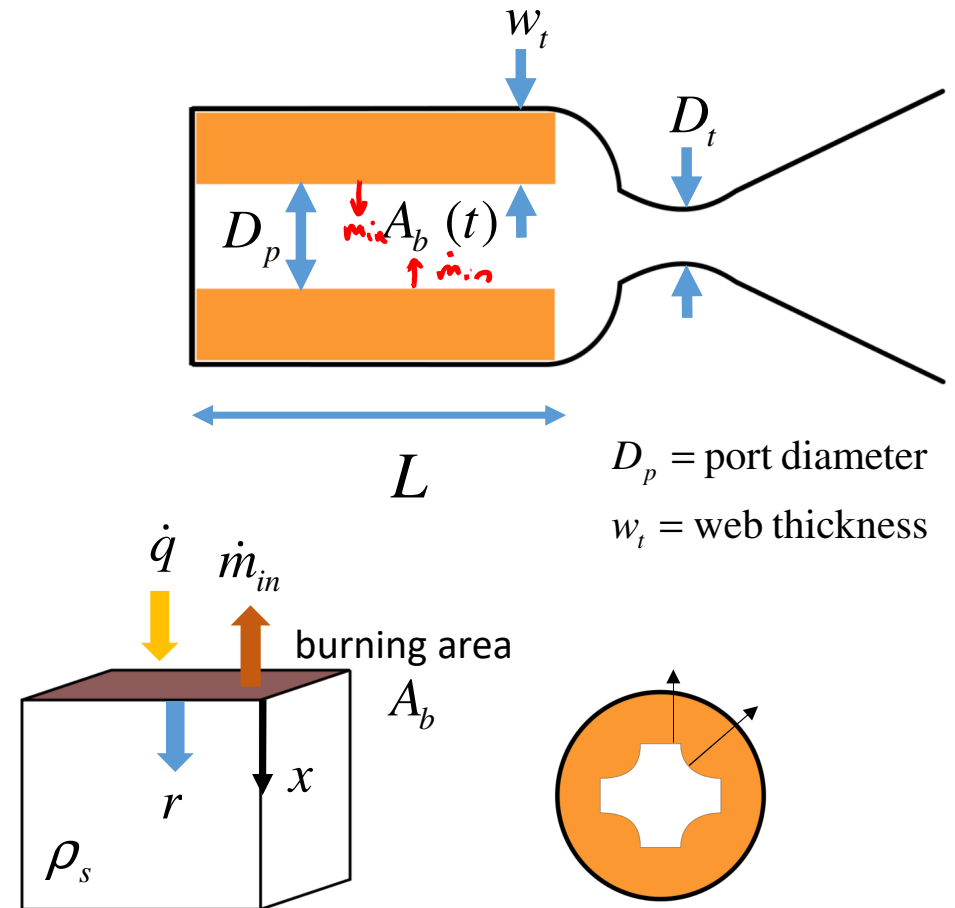
given an initial grain geometry, how do we calculate temporal profiles?

# Solid-rocket motor (SRM)

## Regression history

- want to know local  $dx/dt$  (or  $r$ )
  - $x$  is normal to local surface
- sharp corners smoothed over time
- simplest analysis
  - assume  $p_o$  uniform in port
  - quasi-steady burning,  $p_o \sim$  constant over small amount of regression

rate of change of mass  $\dot{m}_{in} = \rho_s r A_b$



# Solid-rocket motor (SRM)

## Regression history

$a$  = burn rate coefficient/propellant regression rate coefficient  
 $n$  = pressure exponent/propellant regression rate parameter  
 $p_c$  = combustion chamber pressure

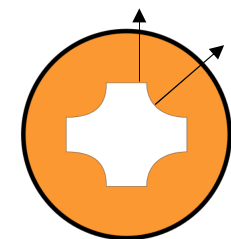
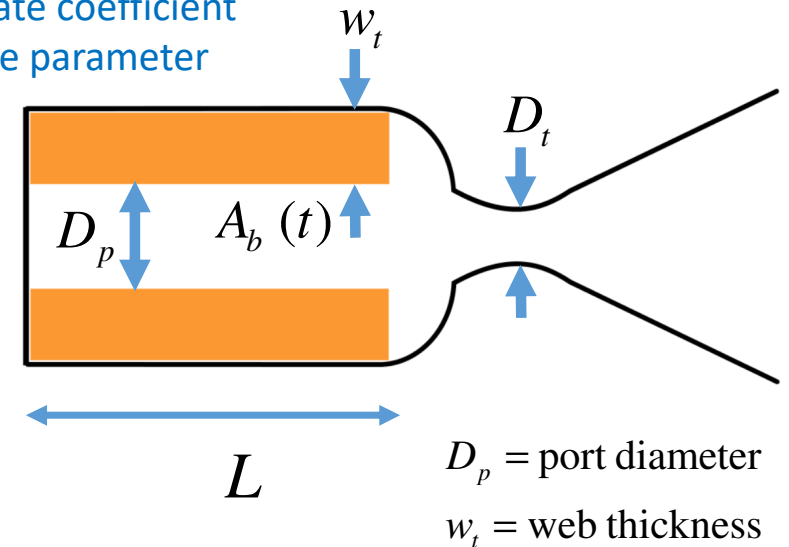
$$p_o \equiv [aK(\rho_s - \rho_o)c^*]^{1/1-n} \quad \text{where} \quad A_b/A_t \equiv K$$

$$r = ap_o^n \quad \Rightarrow \quad \frac{dx}{dt} = r = a \left[ a \frac{A_b(t)}{A_t} \rho_s c^* \right]^{n/1-n}$$

$$r = a [p_o (\text{MPa})]^n \frac{\text{cm}}{\text{s}} \quad *$$

$$\text{so } \frac{dx}{dt} = r = a \left( \frac{\text{cm}}{\text{s}} \right) \left[ \frac{a}{100} \left( \frac{\text{m}}{\text{s}} \right) \frac{A_b(t)}{A_t} \rho_s c^* \frac{\text{MPa}}{10^6 \text{ Pa}} \right]^{n/1-n}$$

always check units, e.g. if burn rate constant specified as  $a = \text{cm/s/MPa}^n$



# Solid-rocket motor (SRM)

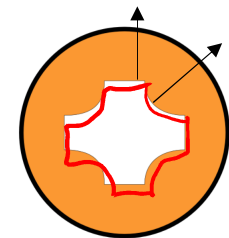
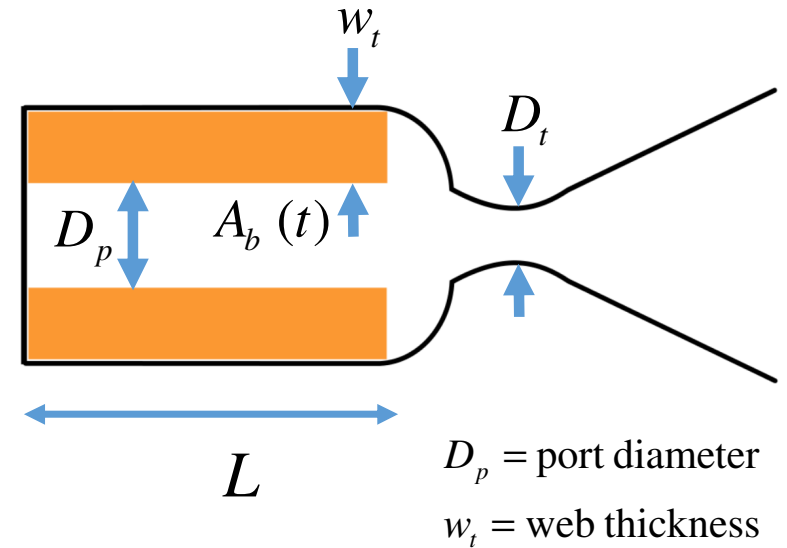
## Regression history

$$\frac{dx}{dt} = r = a \left[ a \frac{A_b(t)}{A_t} \rho_s c^* \right]^{n/1-n}$$

$$\frac{dx}{dt} = r = a \left[ a \frac{S(t)L}{A_t} \rho_s c^* \right]^{n/1-n}$$

- find local  $x(t)$  by integrating along instantaneous normal

$$\int_{x_i}^x \frac{dx}{S^{1-n}} = \int_0^t a^{1/n} \left[ \frac{L}{A_t} \rho_s c^* \right]^{n/1-n} dt$$



# Solid-rocket motor (SRM)

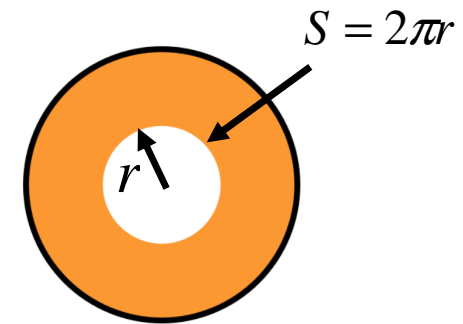
## Regression history

$$\int_{x_i}^x \frac{dx}{S^{\frac{n}{1-n}}} = \int_0^t a^{\frac{1}{1-n}} \left[ \frac{L}{A_t} \rho_s c^* \right]^{\frac{n}{1-n}} dt$$

for an internal burning tube,

$$\int_{r_i}^r \frac{dr}{(2\pi r)^{\frac{n}{1-n}}} = \int_0^t a^{\frac{1}{1-n}} \left[ \frac{L}{A_t} \rho_s c^* \right]^{\frac{n}{1-n}} dt$$

$$\int_{r_i}^r \frac{1}{r^{\frac{n}{n-1}}} dr = (2\pi)^{\frac{n}{1-n}} a^{\frac{1}{1-n}} \left[ \frac{L}{A_t} \rho_s c^* \right]^{\frac{n}{1-n}} t$$



also notice from the form we derived earlier

$$p_o = \left[ aK(\rho_s - \rho_o) c^* \right]^{\frac{1}{1-n}}$$

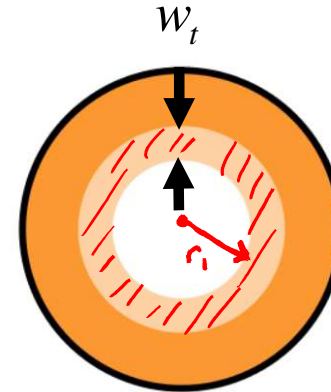
$$p_o \cong \left[ a \frac{2\pi r L}{A_t} \rho_s c^* \right]^{\frac{1}{1-n}}$$

# Solid-rocket motor (SRM)

## Iteration to determine pressure history

$$w_t = \int_0^{t_b} r(t) dt$$

- for cylindrical burning surface  $A_b = 2\pi L(r_i + w_t)$   
 $r_i =$  initial port radius



- generalized expression for chamber pressure

$$p_o = \left[ aK(\rho_s - \rho_o)c^* \right]^{1/n} \quad \text{where } A_b/A_t \equiv K$$

- burn rate a function of time (pressure a function of time), so solve numerically

- calculate  $p_o$  for  $w_t = 0$  (initial web distance)

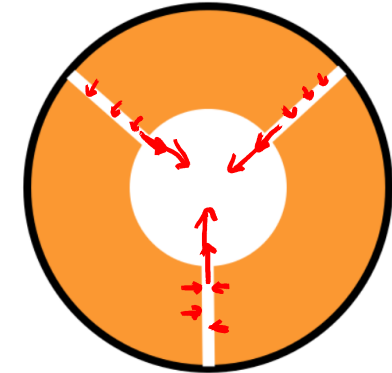
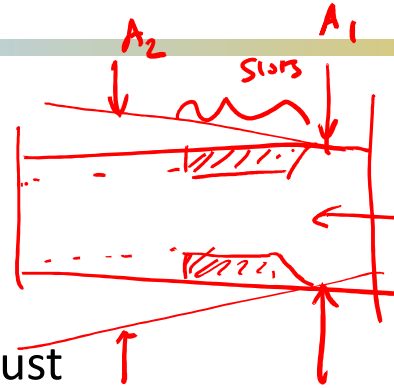
$$\Delta w_t = r \Delta t \quad \text{i.e. Euler integration, with } p_o \text{ and } r \text{ constant over small interval}$$

- calculate new web distance (initial +  $\Delta w_t$ )
- calculate pressure for this new web distance; iterate until web distance = total web distance

# Solid-rocket motor (SRM)

## Slotted grain geometry

- some reasons to use this geometry
  - ease of manufacturing
  - increase of surface area, conditioning thrust
- burning features
  - mass flow from slots: constriction in main flow
  - pressure drop in main flow, increase in pressure at ends of tube
- change in stagnation pressure along tube due to slot



$$\Delta p_o = \frac{1}{2} \rho V_1^2 \left( 1 - \frac{A_1}{A_2} \right)^2$$

$V_1$  = velocity upstream of slot

$A_1$  = port area upstream of slot

$A_2$  = port area downstream of slot

valid for sudden expansion

$$A_2 > A_1$$

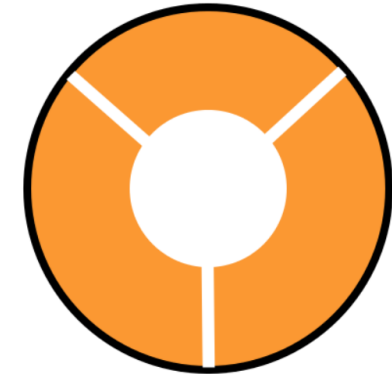
# Solid-rocket motor (SRM)

## Slotted grain geometry

- alternative form

$$\Delta p_o = 0.4 f(M_2) \frac{1}{2} \rho V_2^2 \left( 1 - \frac{A_2}{A_1} \right)^2$$

valid for sudden contraction  
 $A_2 < A_1$



$f(M_2)$  = empirically - determined compressibility correction

$V_2$  = velocity downstream of slot

$$f(M_2) = 1 + 3.05 M_2^4$$

- these relations + conservation relations (mass, momentum) + mass flow from slot:
  - conditions downstream of the slot can be determined
- more complicated analysis: computational fluid dynamics simulations